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ON THE STRUCTURE OF A MORSE FORM FOLIATION

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Abstract. The foliation of a Morse form ω on a closed manifold M is considered. Its maximal components (cylinders formed by compact leaves) form the foliation graph; the cycle rank of this graph is calculated. The number of minimal and maximal components is estimated in terms of characteristics of M and ω. Conditions for the presence of minimal components and homologically non-trivial compact leaves are given in terms of $\operatorname{rk}\omega$ and $\operatorname{Sing}\omega$. The set of the ranks of all forms defining a given foliation without minimal components is described. It is shown that if ω has more centers than conic singularities then $b_1(M)=0$ and thus the foliation has no minimal components and homologically non-trivial compact leaves, its folitation graph being a tree.

Keywords: number of minimal components, number of maximal components, compact leaves, foliation graph, rank of a form

MSC 2000: 57R30, 58K65

1. Introduction and announcement of the results

Consider a connected closed oriented manifold M with a Morse form ω, i.e., a closed 1-form with Morse singularities—locally the differential of a Morse function. The set of its singularities $\operatorname{Sing}\omega$ is finite. This form defines a foliation $\mathcal{F}_\omega$ on $M\setminus\operatorname{Sing}\omega$. Its leaves γ can be classified into compact, *compactifiable* ($\gamma\cup\operatorname{Sing}\omega$ is compact), and non-compactifiable.

Such foliations have remarkably regular structure. A connected component $\mathcal{C}_i^{\max}$ of the union of compact leaves—which we call *maximal component*—is an open cylinder over any its leaf, whose levels are leaves. In particular, all leaves in a maximal component are diffeomorphic. A connected component $\mathcal{C}_i^{\min}$ of the union of non-compactifiable leaves is called *minimal component*. Its topology can be arbitrarily complex—say, such a component can cover the whole $M\setminus\operatorname{Sing}\omega$ [1]—but it cannot be too simple: a minimal component contains at least two cycles with non-